

Spin Physics

R.D. Ball^{†§} and H.A.M^cL. Tallini[‡]

[†]Department of Physics and Astronomy, University of Edinburgh,
Edinburgh EH9 3JZ, Scotland

[‡]Department of Physics, University of Liverpool,
Liverpool L69 3BX, England

Abstract. We review the current situation in polarized scattering experiments. We describe the theoretical interpretation of inclusive deep inelastic processes, the current experimental situation and perturbative analyses which extract structure functions and parton distribution functions. We also discuss various issues such as positivity constraints, small x and the possibility of polarized colliding beam experiments at HERA, semi-inclusive processes, the separation of flavors and the measurement of the gluon polarization, and the possibility of using polarisation experiments to put constraints on new physics.

1. Introduction

In a polarized DIS experiment, both target and beam spins are longitudinally polarized. Analogously to writing the spin averaged DIS cross section in terms of two unpolarized structure functions, the spin dependent differential cross section can be written in terms of two spin structure functions, $g_1(x, Q^2)$ and $g_2(x, Q^2)$:

$$\frac{d^2\sigma^+}{dxdy} - \frac{d^2\sigma^-}{dxdy} = \frac{8\pi\alpha^2 ME}{Q^4} \left\{ \left(2y - y^2 - \frac{Mxy^2}{E} \right) 2xg_1 - \frac{4M}{E} x^2 y g_2 \right\}, \quad (1)$$

where $d^2\sigma^\pm$ are the differential cross sections for target and beam spins respectively parallel and antiparallel. The longitudinal inclusive asymmetry is defined as

$$A_1 = \frac{d^2\sigma_{\gamma N}^+ - d^2\sigma_{\gamma N}^-}{d^2\sigma_{\gamma N}^+ + d^2\sigma_{\gamma N}^-} = \frac{g_1 - \gamma^2 g_2}{F_1} = (1 + \gamma^2) \frac{g_1}{F_1} - \gamma A_2. \quad (2)$$

As the kinematic factor $\gamma = 2Mx/\sqrt{Q^2}$ is usually small, and A_2 is bounded by \sqrt{R} , the longitudinally polarized structure function can be approximated by $g_1 \approx A_1 F_1$.

Just as the unpolarized structure function F_1 has a simple interpretation in the parton model as the sum over the contributions of the two helicity states of quarks and antiquarks,

$$F_1(x) = \frac{1}{2} \sum_i e_i^2 (q_i^+ + \bar{q}_i^+ + q_i^- + \bar{q}_i^-) = \frac{1}{2} \sum_i e^2 q_i(x), \quad (3)$$

where q_i are the unpolarized quark plus antiquark distribution functions of flavour i in the nucleon, so the polarized structure function g_1 is interpreted as the difference,

$$g_1(x) = \frac{1}{2} \sum_i e_i^2 (q_i^+ + \bar{q}_i^+ - q_i^- - \bar{q}_i^-) = \frac{1}{2} \sum_i e^2 \Delta q_i(x), \quad (4)$$

[§]Royal Society University Research Fellow

where $\Delta q_i(x)$ are the polarized quark plus antiquark distributions.

In perturbative QCD, this identification of parton distributions with structure functions is modified by perturbative corrections:

$$\begin{aligned} F_1(x) &= \frac{1}{2}\langle e^2 \rangle [C^{\text{NS}} \otimes q_{\text{NS}} + C^{\text{S}} \otimes q_{\text{S}} + C^{\text{g}} \otimes g] + O(1/Q^2) \\ g_1(x) &= \frac{1}{2}\langle e^2 \rangle [\Delta C^{\text{NS}} \otimes \Delta q_{\text{NS}} + \Delta C^{\text{S}} \otimes \Delta q_{\text{S}} + \Delta C^{\text{g}} \otimes \Delta g] + O(1/Q^2) \end{aligned} \quad (5)$$

where $q_{\text{NS}}, \Delta q_{\text{NS}}$ are flavor nonsinglet combinations of unpolarized and polarized quark plus antiquark densities, $q_{\text{S}}, \Delta q_{\text{S}}$ the corresponding flavor singlet quark densities, and g and Δg the unpolarized and polarized gluon densities. These are convoluted in the usual way with the quark coefficient functions $C^{\text{NS}}, C^{\text{S}}, \Delta C^{\text{NS}}, \Delta C^{\text{S}} = 1 + O(\alpha_s)$ and gluon coefficient functions $C^{\text{g}}(Q^2), \Delta C^{\text{g}}(Q^2) = O(\alpha_s)$, derived from hard cross-sections expanded perturbatively (at NLO) in $\alpha_s(Q^2)$. The parton densities are evaluated at scale Q^2 : their evolution with $t = \ln Q^2$ is then given by the Altarelli-Parisi equations

$$\begin{aligned} \frac{d}{dt} q_{\text{NS}} &= P_{qq}^{\text{NS}} \otimes q_{\text{NS}}, \\ \frac{d}{dt} \begin{pmatrix} q_{\text{S}} \\ g \end{pmatrix} &= \begin{pmatrix} P_{qq}^{\text{S}} & P_{qg}^{\text{S}} \\ P_{gq}^{\text{S}} & P_{gg}^{\text{S}} \end{pmatrix} \otimes \begin{pmatrix} q_{\text{S}} \\ g \end{pmatrix}, \end{aligned} \quad (7)$$

$$\begin{aligned} \frac{d}{dt} \Delta q_{\text{NS}} &= \Delta P_{qq}^{\text{NS}} \otimes \Delta q_{\text{NS}}, \\ \frac{d}{dt} \begin{pmatrix} \Delta q^{\text{S}} \\ \Delta g \end{pmatrix} &= \begin{pmatrix} \Delta P_{qq}^{\text{S}} & \Delta P_{qg}^{\text{S}} \\ \Delta P_{gq}^{\text{S}} & \Delta P_{gg}^{\text{S}} \end{pmatrix} \otimes \begin{pmatrix} \Delta q^{\text{S}} \\ \Delta g \end{pmatrix}, \end{aligned} \quad (8)$$

where the various unpolarized and polarized splitting functions are expanded perturbatively: at NLO $P, \Delta P = O(\alpha_s) + O(\alpha_s^2)$. Note that unpolarized and polarized, and nonsinglet and singlet distributions, evolve independently, while the singlet quark distributions mix with the corresponding gluon distributions.

All of the coefficient functions and splitting functions have now been computed at NLO, the polarized calculations being completed only recently [1]. These calculations were performed using $\overline{\text{MS}}$ subtraction, with a particular definition of γ^5 , and with a further finite subtraction to ensure conservation of first moments (see [1] for details). Of course one can however use them to do calculations in any scheme one chooses by making a NLO redefinition of parton densities. The choice of scheme is of particular relevance for the first moments of the polarized densities.

The first moment of g_1 for proton and neutron targets can be written (assuming exact isospin symmetry and ignoring charm for simplicity) as

$$\Gamma_1^{p(n)} = \frac{1}{12} C^{\text{NS}} (\pm a_3 + \frac{1}{3} a_8) + \frac{1}{9} (C^{\text{S}} \Delta \Sigma + C^{\text{g}} \Delta g),$$

where $\Delta \Sigma$, a_3 and a_8 are the first moments of singlet and non-singlet quark plus antiquark distributions,

$$a_3 = \Delta u - \Delta d, \quad a_8 = \Delta u + \Delta d - 2\Delta s, \quad \Delta \Sigma = \Delta u + \Delta d + \Delta s, \quad (9)$$

Δg is the first moment of the polarized gluon distribution, and $C_{\text{S}}, C_{\text{NS}}$ and C^{g} are the first moments of the corresponding coefficient functions. In a general scheme the first moments of the quark distributions Δq_i will depend on renormalization scale. If we wish to identify them with the quark helicities, it is necessary to adopt schemes in which they are renormalization group invariant [2, 3]: this is possible only if each of

a_3 , a_8 and $\Delta\Sigma$ is scale independent. In all such schemes it can be shown [2] that the first moment of the gluon coefficient function $C^g = -n_f \frac{\alpha_s}{2\pi} C^S$, whence

$$\Gamma_1^{p(n)}(Q^2) = \frac{1}{12} C^{\text{NS}}(\alpha_s(Q^2)) (\pm a_3 + \frac{1}{3} a_8) + \frac{1}{9} C^S(\alpha_s(Q^2)) a_0(Q^2), \quad (10)$$

where the axial singlet charge

$$a_0(Q^2) = \Delta\Sigma - n_f \frac{\alpha_s(Q^2)}{2\pi} \Delta g(Q^2). \quad (11)$$

Because of the Adler-Bardeen nonrenormalization theorem reasonable schemes exist in which these equations hold to all orders in perturbation theory [4]: such schemes have become known as ‘AB schemes’.[†] Naively one might have thought that the second term in eqn.11 is small asymptotically, but this is not the case because $\Delta g(Q^2)$ grows linearly with $\ln Q^2$ [2]. It follows that for first moments of polarized distributions the partonic identification eqn.4 is unjustified; polarized gluons have an effectively pointlike coupling to polarized virtual photons through the triangle anomaly.

The Bjorken sum rule [6] follows directly from eqn.10 and the observation that in the schemes in which a_3 is scale independent it may be identified with the forward nucleon matrix element of the (partially conserved) axial current, and thus with the ratio axial coupling g_A measured in neutron β -decay:

$$\Gamma_1^p - \Gamma_1^n = \frac{1}{6} \left(1 - \frac{\alpha_s}{\pi} - \dots \right) g_A. \quad (12)$$

It is thus a fundamental prediction of perturbative QCD.

Adopting the more dangerous assumption that the strange quarks in the nucleon are unpolarized, and moreover that the gluon polarization is sufficiently small that the distinction eqn.11 between $\Delta\Sigma$ and $a_0(\infty)$ may be ignored, leads to the ‘Ellis-Jaffe sum rule’ [8],

$$\Gamma_1^{p(n)} = \pm \frac{1}{12} \left[1 - \frac{\alpha_s}{\pi} + \frac{5}{3} \frac{3F-D}{F+D} \left(1 - \frac{7\alpha_s}{15\pi} \right) + O(\alpha_s^2) \right] g_A, \quad (13)$$

where the ratio F/D is deduced from hyperon decays[9]. It was the observation by EMC of the violation of this sum rule, from which it was deduced that the axial singlet charge a_0 was much smaller than the octet charge a_8 , that rekindled interest in polarized DIS in 1987 [10]. Two distinct interpretations of this result were suggested: either Δs is unexpectedly large (and negative) but Δg is small so that $a_0 \simeq \Delta\Sigma \ll a_8$, or Δs is indeed small, but Δg is large (and positive) so that $a_0 \ll \Delta\Sigma \simeq a_8$. The basic question still remains as to how the spin of the nucleon is divided up among its constituents:

$$\frac{1}{2} = \frac{1}{2} (\Delta u + \Delta d + \Delta s + \dots) + \Delta g + L_z, \quad (14)$$

where the scale dependence of L_z , the contribution from orbital angular momentum [11], precisely cancels that of Δg . Since the EMC results were published, several second generation experiments were proposed to make more accurate measurements in polarized DIS, and most of these now have results.

2. Experimental programmes and Inclusive Asymmetries

A summary of the data taken by all polarized DIS experiments to date is presented in table 1. The experimental programmes are complementary. SMC uses the highest

[†] An example of such a scheme was constructed in ref.[4]: others are provided by using jets to define the polarized gluon distribution ref.[2], provided sufficient care is taken to absorb all soft contributions into the parton densities [5].

Table 1. Summary of polarized DIS data taken to date.

Expt	Beam E	Target	x-Bjorken
E80(1975) [12]	16 GeV e^-	H-butanol	0.1–0.5
E130(1980) [13]	16-23 GeV e^-	H-butanol	0.2–0.7
E142(1992) [14]	19-26 GeV e^-	^3He	0.03–0.6
E143(1993) [15]	10-29 GeV e^-	NH_3/ND_2	0.03–0.8
E154(1995) [16]	49 GeV e^-	^3He	0.014–0.7
E155(1997) [17]	49 GeV e^-	NH_3/LiD	0.014–0.85
EMC(1985) [10]	100-200 GeV μ^+	NH_3	0.005–0.75
SMC(1992) [18]	100 GeV μ^+	D-butanol	0.003–0.7
SMC(1993) [19]	190 GeV μ^+	H-butanol	0.003–0.7
SMC(1994) [20]	190 GeV μ^+	D-butanol	0.003–0.7
SMC(1995) [20]	190 GeV μ^+	D-butanol	0.003–0.7
SMC(1996) [21]	190 GeV μ^+	NH_3	0.003–0.7
HERMES(1995) [22]	28 GeV e^+	^3He	0.02–0.7
HERMES(1996)	28 GeV $^+$	H	0.02–0.7
HERMES(1997) [23]	28 GeV $^+$	H	0.02–0.7

energy, though least intense, beam and hence has the largest range in x and Q^2 . This large kinematic reach is vital for the measurement of first moments, where the uncertainty of the polarized structure function extrapolation at low x is large. The SLAC experiments do not get to x values as low as SMC, but the high intensity linear accelerator they use allows them to produce results with excellent precision.

Currently HERMES is the only experiment taking data. It uses a novel experimental technique of gaseous polarised targets in an electron (positron) storage ring. It also has excellent particle identification capability which enables it to make a broad range of semi-inclusive as well as inclusive measurements. Gas targets are advantageous because they have a large proportion of polarisable nucleons which can be polarised to a high degree. Though pure neutron targets do not exist, a model of a neutron can be used instead, for example ^3He or deuterium.

Most of the older datasets are displayed in figure 1. The HERMES, SMC and E143 results on g_1^p are compared in figure 2, and are seen to be consistent. Currently the most accurate inclusive data comes from the E155 experiment, whose preliminary results for proton and deuterium targets are shown in figure 3. These results represent a fraction of the data, with data at high Q^2 still being analysed.

3. The Extraction of Polarized Parton Distributions

The analysis of polarized structure function data using NLO perturbative QCD proceeds along just the same lines as that of unpolarized data. An ansatz for the polarized distributions at a starting scale Q_0^2 is chosen (in this case Δq_{NS} , Δq_{S} and Δg), this is evolved up to the data using the evolution equations eqn.8, g_1^N is calculated using eqn.6, and compared to the data. The initial parameterization is then tuned in order to find the best fit. Such calculations have been performed by various groups over the last few years [4, 7, 24, 25, 26, 27].

Here we will describe the results of the recent SMC analysis [7]. This uses the code developed in [4, 24], supplemented by a proper treatment of experimental systematic errors. All except the most recent HERMES and E155 data in figures 2 and 3 was

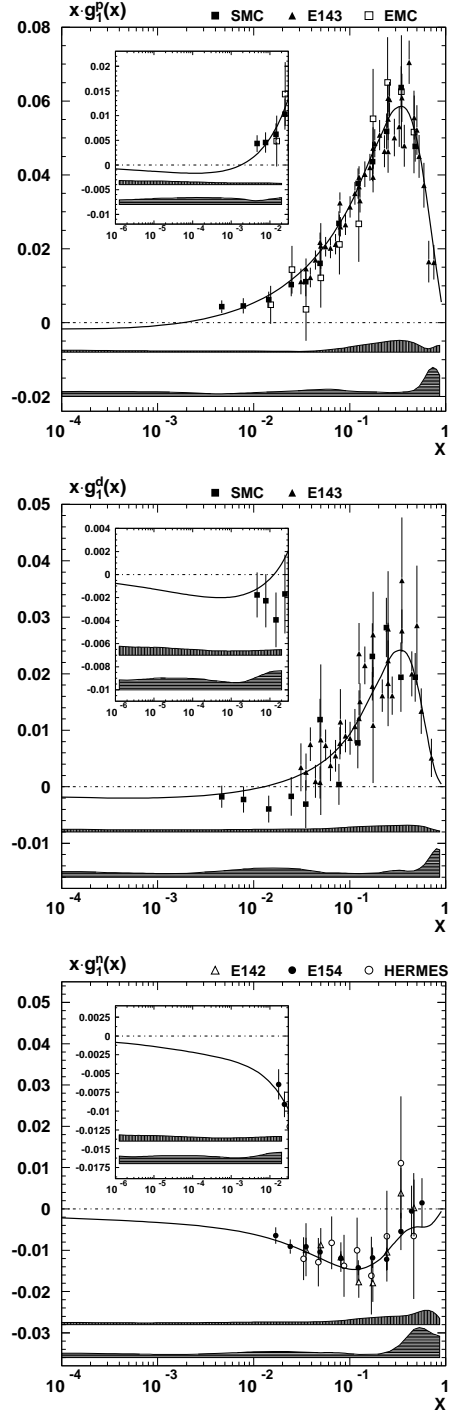


Figure 1. Data on g_1^p , g_1^d and g_1^n . The error bars are statistical: the upper error band gives the systematic errors. Also shown is the global SMC fit [7] at $Q^2 = 5\text{GeV}^2$, with a theoretical error band (lower). The insets show the small x region.

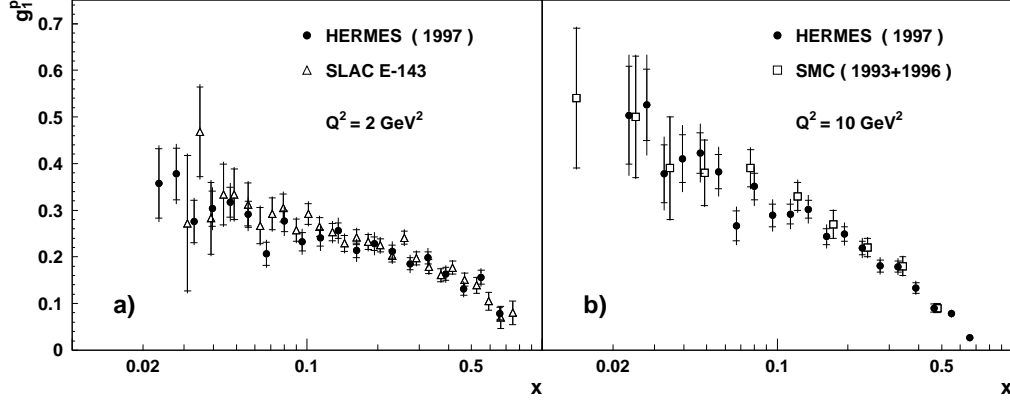


Figure 2. The HERMES g_1^p results evolved to 2 GeV^2 and 10 GeV^2 for comparison with E143 and SMC data.

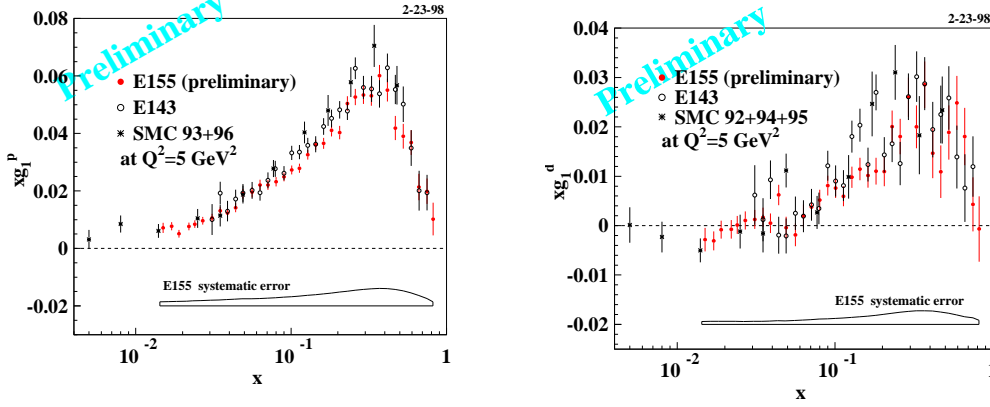


Figure 3. E155 preliminary results on g_1 for (a) the proton and (b) deuteron targets.

included in a global analysis. The fitted curves are displayed in figure 1. The areas under the curves give the first moments of g_1 and from these one can extract (using eqn10)

$$a_3 = 1.20^{+0.08}_{-0.07}(\text{stat.}) \pm 0.12(\text{syst.})^{+0.10}_{-0.04}(\text{th.}) \quad (15)$$

$$a_0(\infty) = 0.24 \pm 0.07(\text{stat.}) \pm 0.19(\text{syst.}). \quad (16)$$

The first of these results confirms the Bjorken sum rule eqn.12 at around the 10% level (since from β -decay $g_A = 1.259$); the second confirms the violation of the Ellis-Jaffe sum rule (since from hyperon-decay $g_8 = 0.58[9]$). Indeed the result for the axial singlet charge is again compatible with zero, just as it was for the original EMC analysis [10]. It is also possible to determine α_s from scaling violations (though not, as yet, from the Bjorken sum rule): the SMC global fit gives

$$\alpha_s(M_Z^2) = 0.121 \pm 0.002(\text{stat.}) \pm 0.006(\text{syst. \& th.}), \quad (17)$$

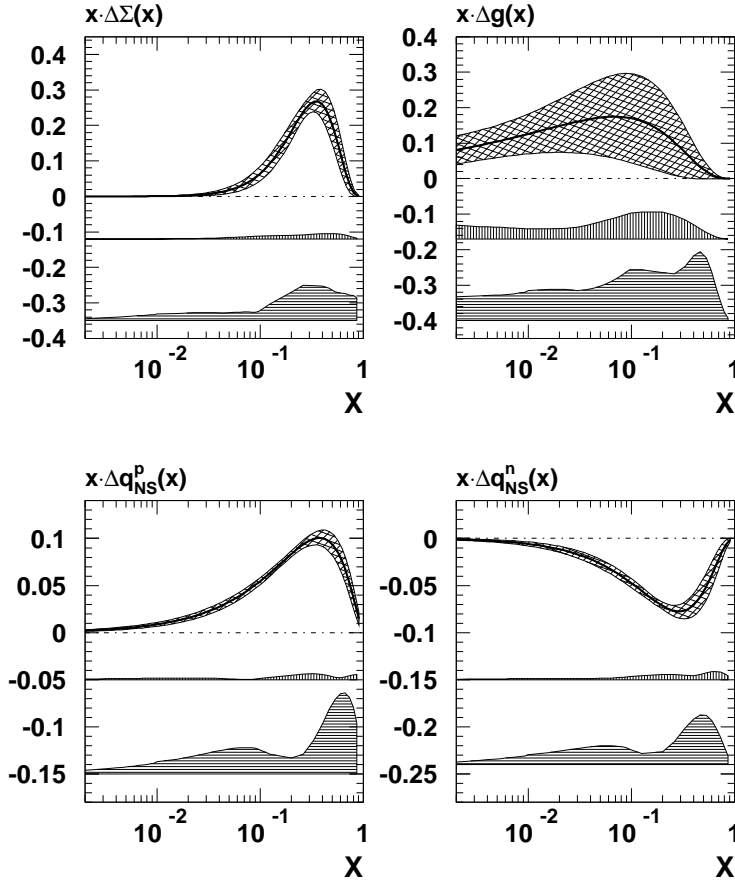


Figure 4. The parton distribution functions $\Delta\Sigma \equiv \Delta q_S$, Δg and Δq_{NS} (for both proton and neutron) obtained from the SMC fits [7]. All the distributions are at $Q_0^2 = 1\text{GeV}^2$ and in the AB-scheme of ref.[4]. The three error bands are statistical, systematic and theoretical respectively.

consistent with an earlier analysis[26] and the world average of 0.118 ± 0.003 .

The polarized parton distributions obtained from these global fits are shown in figure 4. Besides a central distribution, the authors made a full error analysis (something which, incidentally, would also be invaluable in global fits of unpolarized distributions [28]). The error bands in the figures must be treated with caution, since correlations between errors at small and large x are not shown. The theoretical error includes an estimate of NNLO corrections from variation of the renormalization and factorization scales by a factor of $\sqrt{2}$, and an estimate of the dependence on the choice of parameterization. Target mass corrections [29] and higher twist corrections are not included however. For all the distributions the theoretical errors are at least as big as the experimental errors, and at large x are generally much bigger.

From fig. 4 it is clear that both $\Delta q_S(x)$ and $\Delta g(x)$ are significantly positive and of comparable size (though of course the errors on the former are much smaller). This confirms the discovery of a positive Δg first noted in ref.[4]. On the other hand the results obtained for the first moments,

$$\Delta\Sigma = 0.38 \pm 0.03(\text{stat.})^{+0.03}_{-0.02}(\text{syst.})^{+0.03}_{-0.05}(\text{th.}) \quad (18)$$

$$\Delta g = 1.0_{-0.3}^{+1.2}(\text{stat.})_{-0.2}^{+0.4}(\text{syst.})_{-0.5}^{+1.4}(\text{syst.}) \quad (19)$$

suggest that there might also be a sizeable strange quark component (since $\Delta\Sigma$ seems significantly lower than $a_8 = 0.58$).[†]

Comparison of $\Delta q_{\text{NS}}^{\text{p}}$ with $-\Delta q_{\text{NS}}^{\text{n}}$ shows a small discrepancy which can be attributed to the octet contribution (see also ref.[27]). Note however that this discrepancy is considerably smaller than the estimated theoretical error, so it is still difficult [26] to use it to confirm the value of F/D .

4. Positivity Bounds

Since the differential cross-sections eqn.1 are positive, the asymmetry eqn.2 must be bounded by unity because of the triangle inequality: if $d\sigma^{\pm} \geq 0$,

$$0 \leq |d\sigma^+ - d\sigma^-| \leq d\sigma^+ + d\sigma^-, \quad (20)$$

whence $|A_1| \leq 1$. In the parton model the parton distributions are simply proportional to the differential cross-sections (eqns.3,4), and thus the inequality eqn.20 translates into the ‘positivity bound’

$$0 \leq |f^+ - f^-| \leq f^+ + f^-, \quad (21)$$

or $|\Delta f| \leq f$ for each charged parton (so f can correspond to any of the q_i or \bar{q}_i).[‡]

Consider a simplified situation in which there is only one type of parton. Then in NLO perturbative QCD we may write schematically

$$\begin{aligned} d\sigma^+ &= (1 + \alpha_s c^+) \otimes f^+ + \alpha_s c^- \otimes f^-, \\ d\sigma^- &= (1 + \alpha_s c^+) \otimes f^- + \alpha_s c^- \otimes f^+, \end{aligned} \quad (22)$$

where c^+ results from a NLO hard scattering process which preserves helicity, c^- from one that flips helicity. The positivity of the cross-sections now implies that

$$f^+ \geq -\alpha_s c^- \otimes f^- + O(\alpha_s^2), \quad f^- \geq -\alpha_s c^- \otimes f^+ + O(\alpha_s^2). \quad (23)$$

It follows that the partonic positivity conditions $f^{\pm} \geq 0$ or either relaxed or tightened depending on whether the spin flip partonic cross-section c^- is positive or negative. In terms of unpolarized and polarized distributions

$$|\Delta f| \leq 1 + \alpha_s (c - \Delta c) \otimes f + O(\alpha_s^2), \quad (24)$$

where $c = c^+ + c^-$, $\Delta c = c^+ - c^-$. At large scales the NLO bound eqn.24 goes over to the LO bound eqn.21, but conversely at low scales they can be significantly different.

There is nothing mysterious here: partons are not physical objects, but rather scheme dependent theoretical constructions. In a ‘parton scheme’ $c^{\pm} = 0$ and $f^{\pm} \geq 0$ to any order. But in other schemes positivity bounds on parton distributions will generally be nontrivial beyond LO.[§] It follows that it only makes sense to impose

[†] Some caution is required here, however, since the fitted $\Delta q_S(x)$ is strongly valencelike at small x , so the small x uncertainty may be underestimated.

[‡] For usual DIS with virtual photon exchange, the positivity bound is strictly speaking only derivable for the particular parton combinations eqns.3,4. However if we allow chiral exchange currents which couple separately to each flavour, the more restrictive (but equally natural) bounds eqn.21 are required.

[§] Note however that these considerations are independent of the definition of first moments: positivity constraints are of no consequence for first moments since while Δf remains finite, f diverges.

partonic positivity bounds eqn.21 in partonic schemes. The fits which incorporate such bounds [25] thus run the risk of being over-constrained, particularly when they also evolve from scales so low that the use of perturbation theory is unreasonable [26].

A full numerical analysis of NLO positivity bounds requires a physical cross-section which is proportional to Δg at leading order, for example Higgs production in polarized gluon-nucleon scattering, evaluated at NLO [30]. The results confirm that the bounds are only useful at values of $x \gtrsim 0.3$, where the measured asymmetry tends to one. The distributions obtained from the SMC fit [7] lie well inside the bounds, despite the fact that they were not imposed in the analysis. The usefulness of positivity bounds as a constraint on the polarized gluon at very large x is hampered by the large uncertainties in the unpolarized gluon distribution [31], and indeed by the lack of a complete error analysis in global fits to unpolarized data.

5. Polarized Partons at Small x

In order to compute first moments of polarized structure functions or parton distributions, it is necessary to extrapolate the measured distribution from the measured region down to $x = 0$. At one time this was done using Regge theory [32]: this predicts that g_1 will be flat or valencelike at small x , and consequently that the contribution to first moments will be small and under control. However Regge theory is only expected to apply at low Q^2 . Perturbative corrections at larger values of Q^2 generate instabilities due to contributions from logarithms of $1/x$ to the evolution eqns 7,8. For unpolarized distributions the most important small x singularity is that in the triple gluon vertex [33]: this drives a dynamical rise in F_2 at small x which grows as Q^2 increases, which is clearly seen in the data from HERA [34, 35]. In the polarized case all of the splitting functions contain singularities [36], which mix in such a way that g_1 is inevitably driven negative at small x with increasing Q^2 [24, 37]. It follows that when first moments are measured in deep inelastic scattering, the small x contribution is actually rather large and negative, as may be seen from the fitted curves in figure 1. The theoretical uncertainty in the perturbative extrapolation may be conservatively estimated by varying scales: it is inevitably rather large. Thus current estimates of first moments tend to be rather lower than some of the earlier ones, with a larger (and indeed now dominant) uncertainty coming from the small x extrapolation (see table 2).

To reduce this uncertainty it would be very useful to have more data. The current experimental situation on A_1^p at small x is summarised in figure 5. The four points

Table 2. Contributions to first moments of $g_1^N(x, Q_0^2)$ at $Q_0^2 = 5\text{GeV}^2$ [7]. Regge extrapolation of the neutron data makes little sense, as the data themselves show no indication of Regge behaviour.

	$0.003 < x < 0.8$ (meas)	$0.0 < x < 0.003$ (fit)	$0.0 < x < 0.003$ (Regge)
Proton	$0.130 \pm 0.003 \pm 0.005 \pm 0.004$	$-0.012^{+0.014}_{-0.025}$	0.002 ± 0.002
Deuteron	$0.036 \pm 0.004 \pm 0.003 \pm 0.002$	$-0.015^{+0.010}_{-0.023}$	$0.0 \pm 0.005(?)$
Neutron	$-0.054 \pm 0.007 \pm 0.005 \pm 0.004$	$-0.020^{+0.010}_{-0.026}$?

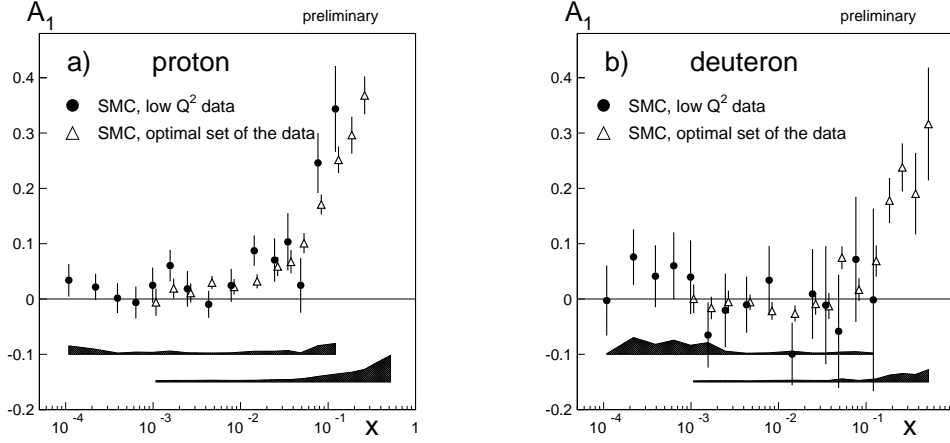


Figure 5. SMC data at small x and low Q^2 [38].

with $x \lesssim 10^{-3}$ have Q^2 between 0.01GeV^2 and 0.2GeV^2 , which should be well inside the Regge region. The results are consistent with a flat or valencelike distribution, very dramatic rises at small x being ruled out, though it is difficult to say much more. Indeed g_1 is very difficult to measure at small x since $A_1 \sim xg_1/F_2$, so even if both g_1 and F_2 were approximately flat (as expected from Regge theory), the asymmetry would still be falling as x , suggesting that at $x \sim 10^{-4}$ it could be smaller than one per mille.

Various studies have been carried out to investigate the exciting possibility of measuring g_1^p at small x at HERA in collider mode [39]. In order to do this the proton beam would have to be polarized: this may be technically feasible, indeed polarized proton beams are being planned for RHIC [40], but is certainly a difficult and costly exercise. Making the optimistic assumption that a luminosity of 500pb^{-1}

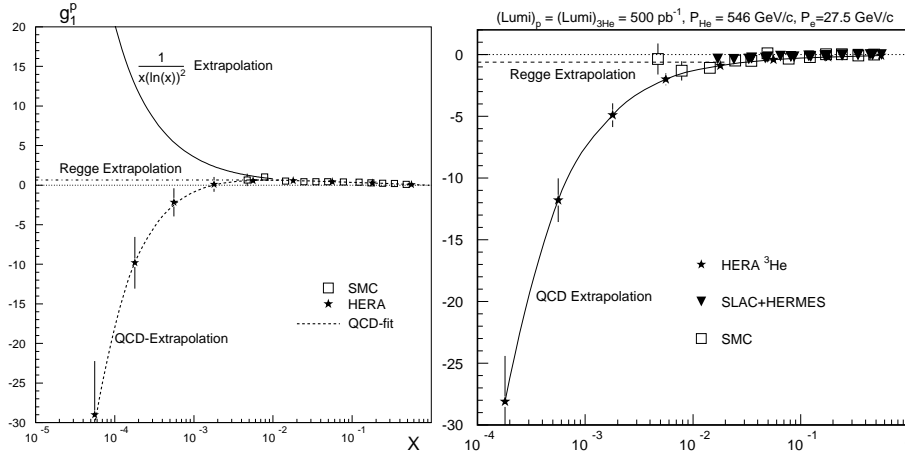


Figure 6. Projected measurements of g_1^p and g_1^d in colliding beam experiments at HERA, assuming a total luminosity of 500pb^{-1} [39].

is obtainable (current luminosities with an unpolarized proton beam are an order of magnitude smaller than this) an idea of the possible accuracy for a measurement of g_1^p is shown in figure 6. Such a measurement would substantially reduce the error in the first moment of g_1^p , and could confirm the general trend of the QCD extrapolation. To improve the precision on the Bjorken sum, necessary if it is to be used to determine α_s at NNLO, and improve the precision on the first moment of Δg , it would be necessary to measure in addition g_1^n . This could be done with a beam of polarized He^3 .

Besides the small x logarithms included in the NLO perturbative calculations described above there are further logarithms of higher orders: in polarized parton evolution these are ‘double’ logarithms of the form $\alpha_s^n \ln^{2n-1} x$. Summing them up gives a very steep perturbative rise [41, 42] (as a power of $1/x$, which in the singlet case threatens to spoil the convergence of first moments). It has been suggested that one could search for such behaviour at a polarized HERA. In practice this would be very difficult: the sign of the powerlike rise is the same as that of the more conventional perturbative behaviour, which it simply reinforces. Double scaling plots [34] would have very few points, with large errors.

In any case our experience in the unpolarized case suggests that summing up small x logarithms in this way is not so easy. Here the double logarithms cancel, leaving only single logarithms of the form $\alpha_s^n \ln^n x$, and it is then possible to extend conventional collinear factorization to all orders even in the small x limit [43]. However recent calculations of the next-to-leading-logarithms [44] (i.e. those of the form $\alpha_s^n \ln^{n-1} x$) show that the expansion in small x logarithms is simply not very useful for values of α_s as large as 0.1 [45]. Various studies of the NLO BFKL equation reveal instabilities leading to negative cross-sections [46]. Given this confusion, it would take considerable courage to put much faith in double logarithm summations in the polarized case.

6. Flavour Decomposition and Semi-inclusive Asymmetries

In semi-inclusive measurements a final state hadron may be used to tag the flavour of the struck quark. Using this extra information, it is possible to measure the separate contributions to the nucleon’s spin from each quark flavour.

A schematic diagram for the process is shown in figure 7. Provided that factorisation holds, the cross section for the production of a particular hadron h is

$$\frac{d\sigma^h(z)}{dz} = \frac{\sum_i e_i^2 q_i(x) D_i^h(z)}{\sum_i e_i^2 q_i(x)} \sigma^T(x), \quad (25)$$

where σ^T is the total inclusive cross section and $D_i^h(z)$ are the fragmentation functions, which represent the probability of a struck quark of flavour i forming a hadron of type h . The spin asymmetry for a hadron h can then be written as

$$A_N^h \equiv \frac{\Delta\sigma_N^h}{\sigma_N^h} = \frac{\sum_i e_i^2 \Delta q_i(x) D_i^h(z)}{\sum_i e_i^2 q_i(x) D_i^h(z)}. \quad (26)$$

The polarized quark distributions can then be extracted from this equation.

Currently only two experiments, SMC and HERMES, are capable of making semi-inclusive measurements and have performed analyses to extract the polarized quark distributions. To maximise the information available, fits are made to positive and negative hadron asymmetries and to inclusive asymmetries for both proton and neutron targets. To reduce the number of unknowns an $\text{SU}(3)$ flavour symmetric

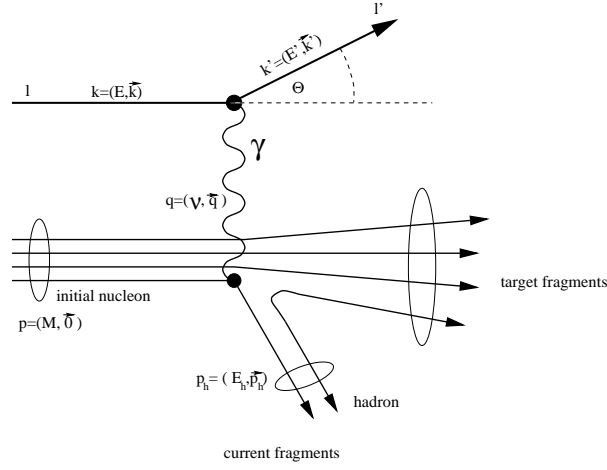


Figure 7. Schematic diagram of a semi-inclusive DIS event.

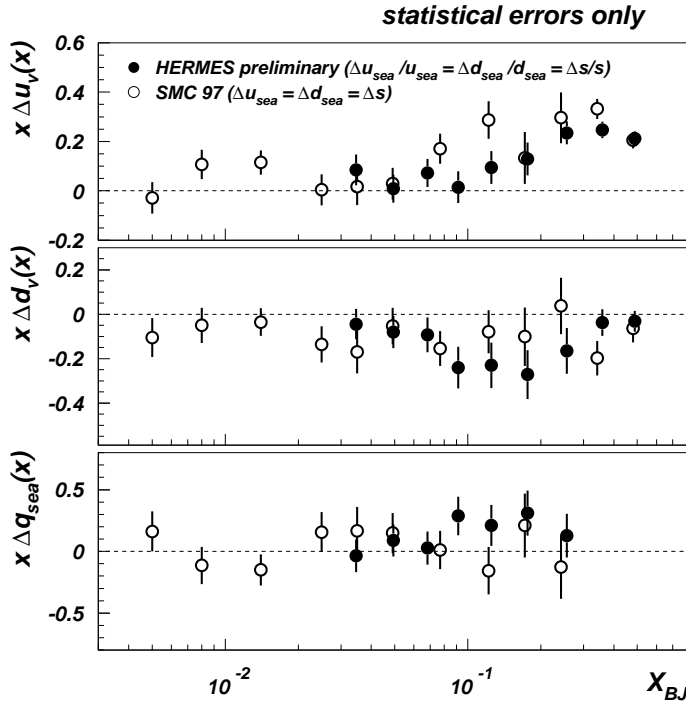


Figure 8. Polarized quark distributions from SMC complete data set [47] and HERMES preliminary results based on the first two years of data taking [48].

sea is assumed. The unpolarized parton distributions are taken from existing parameterisations. An important difference in this semi-inclusive analysis compared to an inclusive analysis is that information about the fragmentation process is required, introducing an extra uncertainty.

Results for the SMC final data set and preliminary results for the first two years of HERMES running is shown in figure 8. The expected difference of sign between up and down valence distributions is confirmed, while the sea distributions are as yet still consistent with zero. NLO perturbative corrections to semi-inclusive processes have been calculated and it is possible to do a NLO global analysis of both inclusive and semi-inclusive data [49].

The HERMES Cerenkov detector was upgraded in 1998 to a ring imaging Cerenkov which will provide hadron identification (i.e. K, p, π separation) over the complete momentum range. While pions are good tags of u and d quarks, kaons are good tags of strange quarks. Thus by using kaon asymmetries in the analysis, it may be possible to remove the constraint of the SU(3) flavour symmetric sea and make a direct measurement of the light and strange sea quark polarisations separately.

An alternative method of determining the flavor decomposition of the proton spin, and furthermore to distinguish cleanly between valence ($q - \bar{q}$) and sea ($q + \bar{q}$) distributions, would be to measure the charged current structure functions

$$\begin{aligned} g_5^{W-} &= \Delta u - \Delta \bar{d} + \Delta c - \Delta \bar{s} - \Delta \bar{b}, \\ g_5^{W+} &= \Delta d - \Delta \bar{u} + \Delta s - \Delta \bar{c} + \Delta b, \end{aligned}$$

at high Q^2 with polarized colliding beams at HERA [50]. The advantages of this method is that a clean NLO analysis is possible; the disadvantage is that as with small x measurements a very high luminosity would be needed to obtain useful results. Another possibility would be to study vector boson production at a hadron collider such as RHIC, with both the beams polarized [40]. Again NLO calculations exist [51] but there are as yet no detailed numerical studies.

It is important to recognise that whatever method one uses, the flavor decomposition of first moments of quark distributions will always be ambiguous due to the (arbitrarily large) ambiguity in the definition of $\Delta\Sigma$. This ambiguity is only resolved if one adopts a scheme in which $\Delta\Sigma$ is a renormalization group invariant [2, 3] (i.e. an AB scheme).

If indeed the axial singlet charge a_0 is suppressed due to the axial anomaly, the effect should be target independent [3]. It might be possible to test this prediction by studying a variety of semi-inclusive processes with polarized colliding beams at HERA [52]. Rather dramatic enhancements are expected in some channels.

7. Direct Measurements of ΔG

Although it is possible to infer something about the polarization of the gluon by the study of scaling violations in inclusive processes, it would clearly be also useful to measure it directly by studying the final state, just as in unpolarized experiments. Photon gluon fusion events are tagged by detecting open charm production, high p_T hadron pairs or (in)elastic J/ψ production. HERMES has upgraded its detector so that it can better measure these channels and is currently taking data. The COMPASS experiment [53] has been approved and is expected to take data from 2001/2. COMPASS has the advantage of having a much higher energy than HERMES

(200 GeV compared to 30 GeV). This not only increases the cross section of charm production, but also reduces the theoretical uncertainty in extracting ΔG from the measured asymmetries.

Perturbative calculations of cross-sections for the photoproduction of heavy quarks have been at NLO [54], and numerical studies [55] suggest that the calculation is under control and that the asymmetries may be large enough to determine Δg if the luminosity is high enough. However it must always be remembered that it is difficult to determine the gluon distribution from charm photoproduction even in the unpolarized case: the charm mass is rather low to be used as a hard scale, and it is difficult to disentangle resolved and unresolved contributions.

A similar technique is to study dijets and high p_T hadrons at a polarized HERA. Preliminary studies at LO [56] show that both these methods could provide a useful measurement of Δg for $x \lesssim 0.1$ provided only that the luminosity is sufficiently high. The NLO corrections [57] appear to be reasonably small.

Complementary experiments in the RHIC spin physics programme [40], or scattering a polarized proton beam at HERA off a polarized fixed target, could determine Δg by measuring double spin asymmetries with prompt photon (or indeed J/ψ) production [58]. At present calculations are only at LO, and there would presumably be problems with intrinsic k_T just as in the unpolarized measurements. However prompt photon measurements will inevitably play an important part in the determination of Δg at large x . Dijets might also be useful, and here there is even a recent calculation of NLO corrections [59].

8. Outlook

The last ten years of polarized scattering experiments have seen some exciting and interesting discoveries, and with COMPASS, a polarized collider at RHIC and possibly eventually HERA, the next ten years look very promising too. Certainly much has, and will, be learnt about perturbative QCD and the distribution of polarized partons in hadrons. However polarizing hadron beams is an expensive business, and at some stage we have to ask whether these sort of experiments are likely to yield anything fundamentally new. Since we know already that the weak interaction is chiral, and thus that any new physics at or around the electroweak scale is likely to be chiral, this may not be so fanciful as it seems. At a polarized RHIC it will be possible to search for chiral contact interactions, a light leptophobic Z' , or right handed charged currents [60]. Particularly interesting in the light of recent developments would be the search for possible chiral contact interactions at a polarized HERA [61]: while with an unpolarized proton beam there are two possible spin asymmetries and four charge asymmetries, when the proton is polarized there are a further twelve spin asymmetries and any number of charge asymmetries. Contact interactions in these channels cannot be explored with unpolarized beams. It thus seems possible that spin might eventually play an important part in the understanding of new physics in the TeV region.

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